TABLE III. Acoustic data for polycrystalline α-Al_2O_3 (at 298°K).

Pressure (kbar)	Density (g/cm³)	Sound velocities v_l (km/sec)	v_t (km/sec)	v_m (km/sec)	Elastic moduli L^s ($\times 10^{11}$ dyn/cm²)	G ($\times 10^{11}$ dyn/cm²)	B^s ($\times 10^{11}$ dyn/cm²)	$\theta_{D(\text{elastic})}$ (°K)
0.001	3.986	10.889	6.398	7.092	47.262	16.316	25.507	1034.9
1.0	3.988	10.894	6.400	7.094	47.333	16.335	25.552	1035.4
2.0	3.989	10.900	6.402	7.097	47.391	16.351	25.590	1035.8
3.0	3.991	10.905	6.404	7.099	47.461	16.370	25.635	1036.4
4.0	3.992	10.910	6.407	7.102	47.520	16.385	25.672	1036.8
5.0	3.994	10.916	6.409	7.104	47.590	16.404	25.718	1037.4
6.0	3.996	10.921	6.411	7.107	47.660	16.424	25.762	1037.9
7.0	3.997	10.927	6.413	7.109	47.720	16.439	25.800	1038.4
8.0	3.999	10.932	6.415	7.112	47.790	16.458	25.845	1038.9
9.0	4.000	10.937	6.418	7.114	47.848	16.474	25.883	1039.4
10.0	4.002	10.943	6.420	7.117	47.919	16.494	25.928	1039.9

from the ultrasonic method and the other, calculated velocities from the measured moduli resulting from the resonant method. Above 300°K, all values of the sound velocities were calculated from measured isotropic elastic moduli and density evaluated at a given temperature.

At 298°K, the isotropic temperature derivatives of sound velocities were: $(dv_l/dT) = -0.37(\pm 0.022)$ and $(dv_t/dT) = -0.29(\pm 0.016)$ in units of 10^{-3} (km/sec)/deg. The corresponding derivatives of the isotropic elastic moduli at 298°K were found as: $(dL^s/dT) = -0.39$ (± 0.030), $(dG/dT) = -0.16(\pm 0.014)$, and $(dB^s/dT) = -0.17$ in units of 10^9 (dyn/cm²)/deg. At 1000°K, however, the corresponding derivatives were as follows: $(dv_l/dT) = -0.53$ and $(dv_t/dT) = -0.41$ in units of 10^{-3} (km/sec)/deg, and $(dL^s/dT) = -0.55(\pm 0.026)$, $(dG/dT) = -0.23(\pm 0.020)$ and $(dB^s/dT) = -0.24$ in units of 10^9 (dyn/cm²)/deg.

Comparing the present data with the similar data available for a Lucalox alumina,[13] we note the following: our values of the adiabatic bulk modulus, for example, at 298°K and also at 1000°K are 25.51 and 23.95 in units of 10^{11} dyn/cm², respectively, whereas the Lucalox data[13] were 24.87 and 23.19 in the same units. The apparent differences of about 3% are beyond the limits of the expected experimental errors in both cases, and these may be associated with the foreign materials contained in the Lucalox specimen.

4. DATA ANALYSIS AND COMPARISON OF POLYCRYSTALLINE DATA WITH THE CORRESPONDING SINGLE-CRYSTAL DATA

A realistic test of the correspondence between the single-crystal acoustic data and polycrystalline acoustic data demands essentially three general requirements: (a) the representative set of the single-crystal acoustic data, (b) the polycrystalline acoustic data evaluated for the corresponding density, and (c) an averaging scheme by which the anisotropic single-crystal acoustic data can be converted into isotropic acoustic data. The requirement (c) has been considered in Ref. 2. The requirement (a) can be met by considering the recent data on the single-crystal elastic constants as a function of temperature[7] as well as a function of pressure.[15] In this section, on the basis of the experimental results presented in Sec. 3, polycrystalline acoustic data corresponding to the single-crystal density are evaluated and compared with isotropic properties calculated from the single-crystal data.

4.1. Variation with Pressure

In the first-order approximation for low porosities, the porosity-dependent elastic modulus M is given by[21,22]

$$M = M_0(1-\alpha\eta), \tag{4.1}$$

where M_0 is the elastic modulus of nonporous materials and η is the porosity. α is a constant. Differentiating Eq. (4.1) with respect to pressure, we find

$$dM/dp = (dM_0/dp)(1-\alpha\eta) - M_0\alpha(d\eta/dp). \tag{4.2}$$

Because the rate of change of porosity with pressure $(d\eta/dp)$ in our specimen is estimated to be -3×10^{-6}/kbar, the last term can be ignored and we obtain

$$(1/M)(dM/dp) = (1/M_0)(dM_0/dp). \tag{4.3}$$

The physical implication of Eq. (4.3) is that the pressure coefficient of an elastic modulus determined on porous polycrystalline aggregate represents the corresponding quantity of the nonporous polycrystalline aggregates. A departure from Eq. (4.3), if observed, would then correspond to *effects* of the $(\alpha\eta)$ term.

On the basis of Eq. (4.3) and experimental results

[21] For a review, see N. A. Weil, in *High Temperature Technology* (Butterworths Scientific Publications Inc., Washington, D.C., 1964), p. 217.

[22] J. B. Walsh, J. Geophys. Res. **70**, 381 (1965).

TABLE IV. Single-crystal elastic constants and their first pressure derivatives for trigonal α-Al_2O_3 (at 298°K).[a]

Index for elastic constants	11	33	44	66	12	13	14
$c_{\mu\nu}^s$ ($\times 10^{11}$ dyn/cm^2)	49.781	50.192	14.752	16.751	16.279	11.735	−2.286
$(dc_{\mu\nu}^s/dp)_{T=298°K}$	6.14	5.03	2.24	1.44	3.26	3.67	0.164

[a] J. H. Gieske (private communications).

presented in Tables I and II, we find from the use of Eq. (3.4) that $(dL^s/dp)=6.57$, $(dG/dp)=1.79$, and $(dB^s/dp)=4.19$. These values represent the first pressure derivatives of the isotropic elastic moduli (evaluated at $p=0$) for the nonporous aggregates of polycrystalline alumina and these are to be compared with the corresponding quantities calculated from the single-crystal data. Table III is a tabulation of the density, sound velocities, and isotropic elastic moduli as a function of hydrostatic pressure. Also entered here are the computed *mean* velocity of sound and the Debye temperature of which a discussion follows later in Sec. 5.

The single-crystal second-order elastic constants and their pressure dependence up to about 10 kbar have been determined recently by Gieske.[15] Gieske's results[15] at 298°K are reproduced in Table IV. Using these single-crystal data, the pressure derivatives of the isotropic elastic moduli were calculated.

The first pressure derivatives of the isotropic polycrystalline elastic moduli in terms of the corresponding single-crystal properties are[2]

$$B^{*\prime}=(B_V'+B_R')/2 \tag{4.4}$$

and

$$G^{*\prime}=(G_V'+G_R')/2, \tag{4.5}$$

where

$$B_V'=[2(c_{11}'+c_{12}')+c_{33}'+4c_{13}']/9 \tag{4.6}$$

$$B_R'=C_b(B_R/C_c)^2C_c'-(B_R^2/C_c)C_b' \tag{4.7}$$

$$G_V'=(C_b'+12c_{44}'+12c_{66}')/30, \tag{4.8}$$

and

$$G_R'=[6B_V(G_R/C_c)^2C_c'-6(G_R^2/C_c)B_V'+4(G_R/C_h)^2C_h'-4(G_R^2/C_h)c_{44}'+2C_a(G_R/C_h)^2C_h'-2(G_R^2/C_h)C_a']/5, \tag{4.9}$$

where

$$C_a=c_{11}-c_{12} \quad \text{and} \quad C_a'=c_{11}'-c_{12}' \tag{4.10}$$

$$C_b=c_{11}+c_{12}+2c_{33}-4c_{13}$$

and

$$C_b'=c_{11}'+c_{12}'+2c_{33}'-4c_{13}' \tag{4.11}$$

$$C_c=c_{33}(c_{11}+c_{12})-2c_{13}^2$$

and

$$C_c'=(c_{11}+c_{12})c_{33}'+c_{33}(c_{11}'+c_{12}')-4c_{13}c_{13}' \tag{4.12}$$

$$C_h=C_ac_{44}-2c_{14}^2$$

and

$$C_h'=C_ac_{44}'+c_{44}Ca'-4c_{14}c_{14}' \tag{4.13}$$

$$B_V=[2(c_{11}+c_{12})+c_{33}+4c_{13}]/9 \tag{4.14}$$

$$B_R=[c_{33}(c_{11}+c_{12})-2c_{13}^2]/(c_{11}+c_{12}+2c_{33}-4c_{13}) \tag{4.15}$$

$$G_R=(5/2)\{(C_cc_{44}c_{66})/[C_c(c_{44}+c_{66})+3B_Vc_{44}c_{66}]\} \tag{4.16}$$

and the primes denote the first pressure derivatives. From $B^{*\prime}$ and $G^{*\prime}$ from Eqs. (4.4) and (4.5), the pressure derivative of the isotropic longitudinal modulus $L^{*\prime}$ can be found as

$$L^{*\prime}=B^{*\prime}+4G^{*\prime}/3. \tag{4.17}$$

The calculated values for $B^{*\prime}$, $G^{*\prime}$, and $L^{*\prime}$ are compared with the polycrystalline acoustic data in Table V. Also entered in the table are the limiting values. It is noted that the calculated and experimental values compare very well.

Using the procedure outlined in Ref. 2, the isothermal pressure derivatives of the adiabatic elastic moduli have been converted into (i) isothermal pressure derivatives of the isothermal elastic moduli and (ii) the adiabatic pressure derivatives of the adiabatic elastic moduli; the results are summarized in Table VI. In

TABLE V. Comparison of predicted and experimental isotropic pressure derivatives of polycrystalline elastic moduli for trigonal α-Al_2O_3 (at 298°K).

Pressure derivatives	Density (g/cm^3)	dB/dp			dG/dp			dL/dp		
		(dB_V/dp)	(dB_R/dp)	(dB^*/dp)	(dG_V/dp)	(dG_R/dp)	(dG^*/dp)	(dL_V/dp)	(dL_R/dp)	(dL^*/dp)
Predicted[a]	3.986	4.28	4.26	4.27	1.63	1.83	1.73	6.45	6.70	6.58
Experimental	3.986	...	...	4.19	...	...	1.79	...	...	6.57

[a] Calculated from the single-crystal acoustic data.